Comparative Analysis of Mode Reflection and Transmission in Presence of a Cutoff Cross Section of Nonuniform Waveguide by Using the Cross Section and the Mode-Matching and Generalized Scattering-Matrix Methods

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***Abstract*—By using the cross-section method, waveguide transitions containing parts where modes are in cutoff are analyzed. The problem of infinite value of coupling coefficients for modes in cutoff has been solved. Formulas for the calculation of reflection coefficients are presented. The results have been compared with the mode-matching and generalized scattering-matrix methods.**

***Index Terms*—Cross-section method, coupling coefficients, cutoff modes, generalized scattering-matrix method, mode-matching method.**

I. Introduction

THE cross-section method [1] is an analytical tool used in the analysis and design of components required for low-loss and highly efficient transmission of electromagnetic waves in nonuniform waveguides. In spite of the fact that the results of many papers published about different versions of this method were applied to practical engineering problems, it has not been completely investigated in detail and few numerical examples are available for cutoff cross sections.

The initial ideas related to this method belong to Kisunko's book [2], but the first paper in western countries was the report of Stevenson [3]. Nevertheless, the intensive application of the method did not start until after the work of Schelkunoff [4]. Intensive research was done in many countries for the development of oversized waveguide communication systems in the millimeter-wave range where multimode concepts and coupled-wave theory played a central role. Sporleder and Unger's book [5] concentrates mainly on the topics of tapers, transitions, and couplers.

With some exceptions, this research has been abandoned when the optical fibers appeared, but the results of the coupled-wave theory are being used today for many applications of integrated optics. An interesting review of coupled-wave theory from the optical point-of-view is presented in [6] where recent controversies are discussed.

On the other hand, in the eastern countries, the activities in the use of the cross-section method have continued since the 1960's, as reported in [1] and the extensive references given there. In [1], the most general formulation of the cross-section method is presented together with many practical devices. Additional recent eastern works are [7] and [8], which deal with transitions of different cross-section waveguides.

The basic idea of this method consists of the fact that the electromagnetic field in a nonuniform waveguide is represented by means of a superposition of the mode fields corresponding to more simple waveguides. It reduces a three-dimensional (3-D) electrodynamic problem to the consideration of both the two-dimensional (2-D) problem of the uniform waveguide modes and the one-dimensional (1-D) problem of the solution of a set of ordinary differential coupled-wave equations.

When the waveguide dimensions are large compared with the wavelength, and the geometry of the problem includes bends and other complex structures, a fully 3-D analysis employing modern numerical methods (finite elements, finite-difference time domains, etc.) is practically impossible, and the cross-section method is the only feasible analysis technique. The kind of sophisticated waveguide devices that are analyzed and designed by means of the cross-section method range from highly oversized mode converters and tapers, bends, twisted waveguides, polarizers, and many others necessary for high-power millimeter-wave applications, as shown in [1].

Moreover, the cross-section method (as the most general to investigate the coupled-wave mechanisms in overmoded wave-

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guides) has been employed to evaluate the amplitudes of surface waves scattered by inhomogeneous sections [9]. More recently, the cross-section method and the concept of local modes (reference waveguide) has been referenced in modern open waveguide applications (see, e.g., [10]).

However, in the case of simple nonuniform waveguide structures having specific symmetry, the so-called mode-matching and generalized scattering-matrix methods can be employed without too high requirements for computer memory and computing time even in oversized problems, and they will be employed in this paper to verify the results derived by the cross-section method. In the case of general nonuniform oversized devices, the application of the mode-matching and generalized scattering-matrix methods is very difficult.

The mode-matching and generalized scattering-matrix methods are explained in detail in [11] and the extensive references given there (see also the comments and references given in [1]). In a few papers, the coupled-wave equations are employed and compared with the mode-matching and generalized scattering-matrix methods for the analysis of circular tapers, as in [12] and [13]. Close agreement between both methods and comparable computation times are demonstrated. Some interesting analyses of waveguide transitions by using the coupled-wave equations and the moment method are reported in [14] and [15].

Recently, hybrid methods have been proposed for the analysis and design of complex waveguide devices, as is described in [16], but these hybrid methods have not been demonstrated in highly oversized conditions.

Generally, if waveguide properties vary slowly, it is easy to solve the coupled-wave equations and to obtain an explicit expression for the calculation of the waves scattered on a nonuniform waveguide section using a method of the Wentzel–Kramers–Brillouin (WKB) type, as described in [17]–[19]. This method was introduced in 1912 by Lord Rayleigh for the solution of wave propagation problems and is based on the approximation of the wavenumber by the first terms of a Taylor series. It can be considered as a high-frequency (geometrical optics) method to obtain approximate solutions. However, if a waveguide contains a cutoff cross section, i.e., a cross section separating at a given frequency, the propagating and cutoff regions of a given mode, the values of the coupling coefficients are not small even for very slowly varying parameters and solutions of the WKB type are not valid. In this case, an equivalent boundary condition substituting the cutoff cross section is derived. This condition allows us to obtain the solution of the coupled-wave equations far away from the cutoff cross section.

A. Cross-Section Method

The basic idea of the cross-section method is that the electromagnetic fields in an arbitrary nonuniform waveguide cross section are represented as a superposition of the waves of different modes propagating in the forward and backward directions along an auxiliary straight uniform waveguide of the same cross section and with identical distribution of ε and μ. The coefficients P_j of this superposition satisfy first-order ordinary differential equations [1]

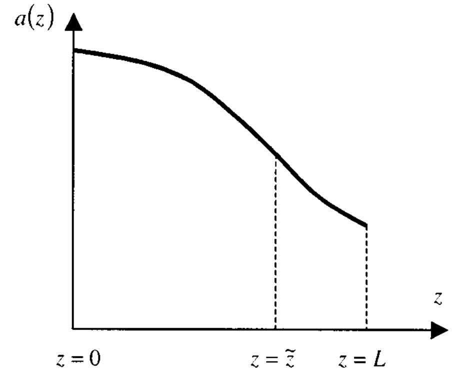


Fig. 1. Schematic of a nonuniform waveguide of arbitrary length L containing a cutoff cross section at $z = \tilde{z}$. The waveguide is axisymmetric with respect to the z-axis.

$$\frac{d}{dz}P_j + ih_jP_j = \sum_{m=-\infty}^{\infty} S_{jm}P_m \tag{1}$$

where z is the direction of propagation, $i = \sqrt{-1}$, P_j, and h_j are, respectively, the amplitude and waveguide wavenumber of the j mode, S_{jm} are the coupling coefficients to the j mode, and a negative value of j denotes a backward propagating mode.

The coupling coefficients depend on the type of nonuniformity under study. In general, every nonuniformity can be reduced to a combination of varying curvature, filling medium, and cross section of the waveguide. For many years, there has been some controversy about the exact formulation of coupling coefficients and coupled-wave theory. One of the most extensive and understandable papers is Doane's [20], where the propagation and coupling of modes is analyzed in smooth and corrugated circular oversized waveguides for the design of very high-power millimeter waveguides. Later, Li and Thumm [21], [22] reported the exact formulation of the coupling coefficients presented in Doane's paper for the case of wall impedance and diameter changes and curvature cases. All these results can be derived from the very general expressions given in [1].

The general problem of the field derivation in a nonuniform waveguide is reduced in such a way to the problem of the fields in a uniform waveguide and to the solution of coupled-wave ordinary differential equations.

II. Equivalent Boundary Condition for Main Modes

In a cutoff cross section of the mode j, located at $z = \tilde{z}$ (see Fig. 1), the coupling coefficients S_{jm} are infinite for all or nearly for all m at $z = \tilde{z}$, and the S_{jm} have large values near $z = \tilde{z}$. Some coefficients in the coupled wave equations (1) for P_j and P_{-j} are infinite at $z = \tilde{z}$ and, hence, solutions of the WKB type are not valid. We will derive a boundary condition for them, which is equivalent to the presence of a cutoff cross section. Further, by applying this boundary condition, one could use these equations and the solutions of the "generalized" WKB type almost everywhere in the nonuniform waveguide.

The reflection of the incident mode at a cutoff cross section means a strong coupling between forward and backward waves.

Near the cutoff cross section, the decomposition of the total field into forward and backward waves does not correspond to the physical nature of the phenomena. Hence, it is inconvenient for our calculations.

The amplitudes P_j and P_{-j} can become infinite for some cross sections. However, only the total fields $\vec{E}$ and $\vec{H}$, and not single terms of their expansions as a sum of the modal fields of the reference waveguide, must have finite values.

To avoid these problems, let us employ instead of the wave amplitudes P_j and P_{-j} the variables Q_j and R_j, related to them by [1]

$$Q_j = P_j - P_{-j}; \quad R_j = P_j + P_{-j}. \tag{2}$$

These variables satisfy the coupled-wave equations

$$Q'_j + ih_j R_j = \sum_{m=1}^{\infty} Q_m \big(S_{jm} - S_{-jm}\big) \tag{3a}$$

$$R'_j + ih_j Q_j = \sum_{m=1}^{\infty} R_m \big(S_{jm} + S_{jm}\big). \tag{3b}$$

There, the total field can be decomposed into TE and TM modes. The coefficients $S_{jm} \pm S_{-jm}$ in (3) have different forms for these two kinds of modes.

To avoid infinite values of the coefficients of these equations, we transform them into second-order differential equations and, taking into account the field of an incident TE mode and the terms of zeroth and first order of the perturbation v only, we obtain

$$Q''_m + h_m^2 Q_m = 0. \tag{4}$$

Here, the value v has the order of the tangent between the z-axis and waveguide wall.

Standard methods like simple WKB, i.e., the methods of "geometrical optics," can be applied to solve coupled-wave equations like (4) in the region, where $|h_m|$ is not small, more exactly, in the region where $|h'_m/h_m^2| \ll 1$. This condition is not valid near the cutoff cross section, thus, we have to derive another solution employing the Airy functions [18], [19]. To do this, we transform (4) into

$$\ddot{Q}_m - tQ_m = 0 \tag{5}$$

where the dot (i.e., $\ddot{}$) denotes derivative with respect to a new variable t, $t = t(z)$ given by

$$t = A^{2/3} k (z - \tilde{z}) \tag{6}$$

with

$$h_m^2 = -k^3 A^2 (z - \tilde{z}) \tag{7}$$

$$A^2 = -\frac{1}{k^3} (h_m^2)' \Big|_{z=\tilde{z}} \tag{8}$$

being k the free-space wavenumber. In our case, $L > \tilde{z}$, $h_m(L)^2 < 0$.

The general solution of this equation is given by a linear combination of the Airy functions

$$Q_m = Mu + Nv \tag{9}$$

where the ratio of the coefficients N and M can be determined by substitution of this solution into the boundary condition $P_{-m}(L) = 0$ and equals

$$\frac{N}{M} = -\frac{\dot{u}(t_L) + \Big(t_L^{1/2} + B\Big) u(t_L)}{\dot{v}(t_L) + \Big(t_L^{1/2} + B\Big) v(t_L)} \tag{10}$$

with

$$B = -\frac{1}{k}(S_{mm} - S_{-mm}) A^{-2/3}.$$

Here, t_L denotes the value of the variable t [see (6)] at $z = L$.

This solution is valid in the same region where (7) can be used, i.e., approximately for $h_m^2 \ll k^2$. This region is overlapped by the region where the condition $|h'_m/h_m^2| \ll 1$ is satisfied, and it can be written in the form $|t| \gg 1$. Therefore, (4) must approach the solutions obtained in the geometrical optics (i.e., WKB) approximation for $|t| \gg 1$. Taking this into account, we use (9) to establish a relationship between P_m and P_{-m} on the boundary of the region $|h'_m/h_m^2| \ll 1$. This relationship can be used as a "boundary condition" [see (13) and (14)] that allows us to limit ourselves to a region far from the cutoff cross section.

III. Phase of Reflection Coefficients at a Cutoff Cross Section

If the mode does not penetrate the narrow waveguide (see Fig. 1), the absolute value of the reflection coefficient is equal to one, and its phase δ, given by the ratio between P_{-m} and P_m, satisfies in the region where geometric optics is applicable the equation

$$\delta' - 2h_m = 0 \tag{11}$$

providing $|h'_m/h_m^2| \ll 1$.

The boundary condition of this equation is obtained taking the asymptotic expression for large negative values of t of the relation between P_{-m} and P_m in the region where (11) is applicable. This relation is given by

$$\exp(i\delta) = \frac{\big[\dot{u} + (t^{1/2} + B)u\big] + (N/M)\big[\dot{v} + (t^{1/2} + B)v\big]}{\big[\dot{u} + (-t^{1/2} + B)u\big] + (N/M)\big[\dot{v} + (-t^{1/2} + B)v\big]}. \tag{12}$$

Thus, the desired reflected phase, which gives the equivalent boundary condition for (1), is given by

$$\delta(0) = 2\int_{\tilde{z}}^{0} h_m(z)dz - \frac{\pi}{2} + \delta_0 = -2\tilde{\gamma}_m - \frac{\pi}{2} + \delta_0 \tag{13}$$

where

$$\exp(i\delta_0) = \frac{(N/M) - i}{(N/M) + i}. \tag{14}$$

The physical meaning of the first term in (13) is obvious. It is the phase acquired during the propagation to the cutoff cross section and back, calculated in the geometric optics approximation. The value δ_0 depends on the distance between the cutoff

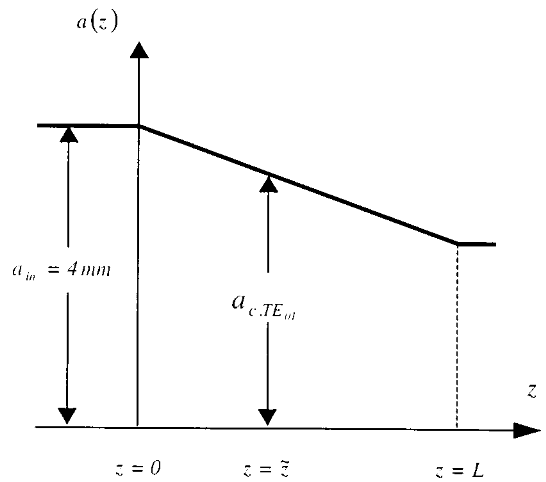


Fig. 2. Schematic showing the circular waveguide transition of arbitrary length L analyzed to study the phase of the reflection coefficient at a cutoff cross section. The waveguide is axisymmetric with respect to the z-axis, the input radius is 4 mm and the cutoff cross-section radius for the TE_{01} mode is a_c.

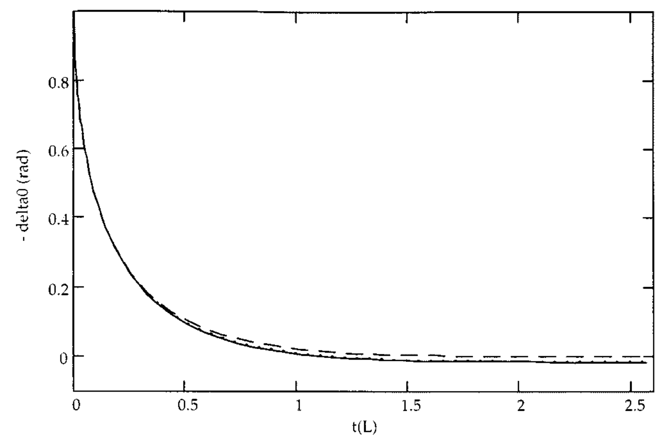


Fig. 3. Comparison between (14) (dashed line), numerical solution of the coupled-wave equations taking into account 1 mode (dotted line), and the mode-matching and generalized scattering-matrix method results (solid line).

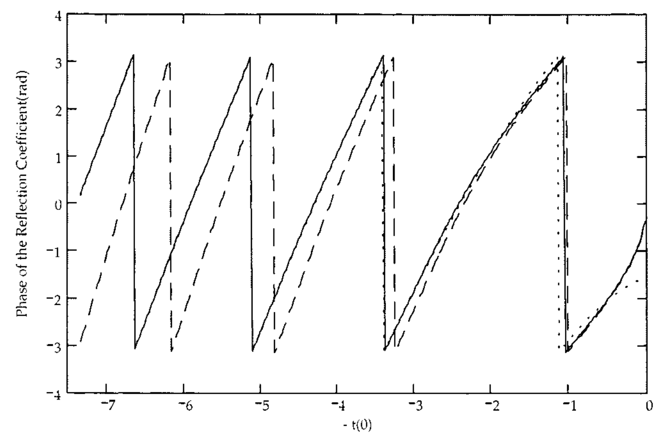


Fig. 4. Phase of the reflection coefficient at the beginning of a nonuniform waveguide containing a cutoff cross section versus the position of the cutoff cross section ($\upsilon = -0.0475$, $t_1 = 2.57$). Comparison between (12) (dashed line), (13) (dotted line), and the mode-matching and generalized scattering-matrix method results (solid line).

cross section and the beginning of the narrow waveguide. It gives an amendment to the phase of the reflected wave caused by the narrow waveguide. Power does not propagate in the narrow waveguide. Nevertheless, the field penetrates to the region $z > \tilde{z}$, being exponentially attenuated along the distance from the cutoff cross section. Thus, the structure of the waveguide in this region influences the phase of the reflected wave. According to (10) and (14), the exact form of the function $\delta_0(t_L)$ depends on the type of mode and on the waveguide properties. For $t_L \gg 1$, $\delta_0 \to 0$ and $\delta(0) = -2\tilde{\gamma}_m - (\pi/2)$.

A similar technique is employed to determine the reflection phase for TM modes, but we do not provide those derivations here. Let us just note the reflection phase of TM modes is equal to $(-2\tilde{\gamma}_m + (\pi/2))$ for $t_L \gg 1$.

In order to prove the validity of (13), a linear taper in a circular waveguide where the TE_{01} mode goes below cutoff (see Fig. 2) has been analyzed using two independent methods: the mode-matching and generalized scattering-matrix methods and the numerical integration of the coupled-wave (1) using the modification proposed in [23] in order to avoid the singularities of the coupling coefficients. In this last case, only one mode has been taken into account, due to the numerical problems caused by the evanescent modes (see [6] and [12]). In fact, this is the same case whose approximate analytical solution is given by (13). The obtained results can be seen in Fig. 3, where the plot of (14) is compared with the numerical results. As can be seen, there is a good agreement between the results of both numerical methods and a little mismatch with the analytical solution. This mismatch is of the order of the error committed by the asymptotic expression used to obtain (13). More detailed considerations show that this mismatch decreases when the transition is smoother.

For a linear transition with $t_L = 2.57$, the evolution of the phase of the reflected wave at the beginning of the waveguide, i.e., at $z = 0$, is given in Fig. 4 as a function of $t(0)$, i.e., of the distance from the origin to the cutoff cross section. The slope of the transition remains constant ($\upsilon = -0.0475$) so that the effect is the same as if the cross section at $z = 0$ is moved to the right-hand side and the cutoff cross section is not moved. As can be seen, the numerical results are in good agreement with those predicted by the theory developed above; for small values of $|t(0)|$ (beginning of the waveguide close to $\tilde{z}$), the numerical results approach (12) and when $|t(0)|$ increases, they are practically identical to (13).

IV. External Critical Cross Section near the End of the Waveguide

Now let us determine the reflection coefficient in the case when the mode penetrates the narrow waveguide. In this case, h_m^2 is positive anywhere and, strictly speaking, there is no cutoff cross section at all. However, the coupling coefficients near the beginning of the narrow waveguide have large values. In such a case, the reflection coefficient can be nearly equal to one. The method developed above can be applied to this problem. One should only let the cutoff cross section be situated outside the taper, i.e., outside the nonuniform waveguide, as it is shown in Fig. 5, thus, $\tilde{z} > L$.

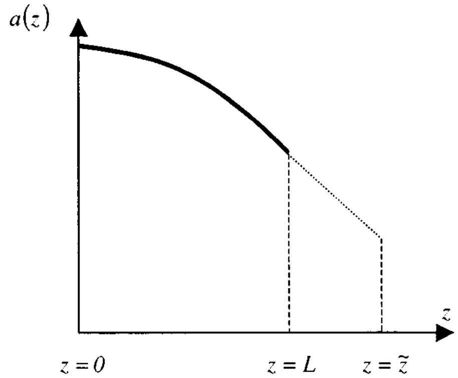


Fig. 5. Schematic showing a transition of arbitrary length L where the propagating mode does not go below cutoff. The waveguide is axisymmetric with respect to the z-axis and the dotted contour indicates a fictitious waveguide where the cutoff is located at $z = \tilde{z}$.

If the mode penetrates the narrow waveguide, i.e., the mode does not go below cutoff, the reflection coefficient, if $|t_L|$ has a large value, is equal to

$$\rho(0) = -\int_0^L S_{-mm} \exp(-2\gamma_m) dz. \tag{15}$$

For low and finite values of $|t_L|$, following [1] a constant term, determined in the same way as the boundary condition given before, must be added. We obtain

$$\rho_0 = \frac{(N/M) - i}{(N/M) + i} \exp\left[-2i(\gamma_m + \alpha_m)\right] \tag{16}$$

where α_m is the eigenvalue of the mode m. The ratio N/M has been given in (10), but in contrast to the previous subsection t_L, has a negative value in this ratio.

To match these two expressions, one should find a second term in the expansion of ρ_0 as a Taylor series of v, where v has again the order of tangent between the z-axis and waveguide wall. In case of a circular conical taper (see Fig. 6) and the TE_{01} mode, a more rigorous value of the reflection coefficient is given by [1]

$$\rho(0) = \rho_0 - \frac{iv}{4\mu_m}\left[\frac{1+\rho_0^2}{g^3} - \rho_0\left(\frac{1}{3g^3} - \frac{1}{g} - \arctan(g)\right)\right] \tag{17}$$

where μ_m is the mth zero of the derivative of the zeroth-order Bessel function and

$$g = \frac{h_m(0)}{\alpha_m(0)}$$

where $\alpha_m(0)$ is the eigenvalue of the mode m at $z = 0$. According to (10), the dependence of ρ_0 on t_L differs for various waveguides. The function $|\rho_0|$ versus $-t_L$ is plotted in Fig. 7 for the TE_{01} mode in circular waveguide and the linear transition shown in Fig. 6, compared with the results obtained for the same transition with the mode-matching and generalized scattering-matrix methods. As can be seen, both results are in very good agreement and, as it is predictable, when the length of the taper approaches $\tilde{z}$, the value of the modulus of the reflection coefficient given by (17) approaches one.

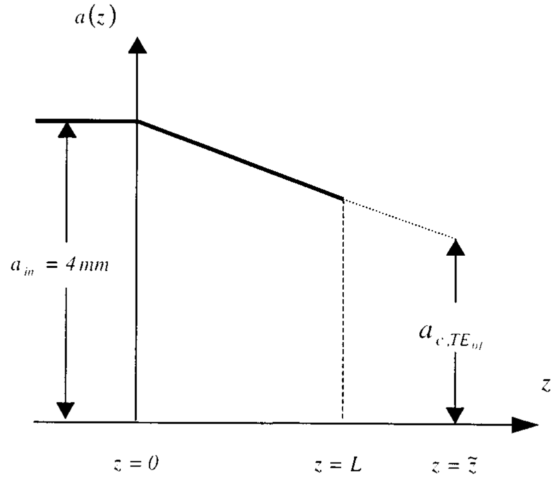


Fig. 6. Schematic showing the circular waveguide transition of length L analyzed to study the reflection coefficient close to a cutoff cross section (the cutoff cross section is outside the waveguide). The waveguide is axisymmetric with respect to the z-axis, the input radius is 4 mm and the cutoff cross-section radius for the TE_{01} mode, a_c, is located in a fictitious waveguide outside the device.

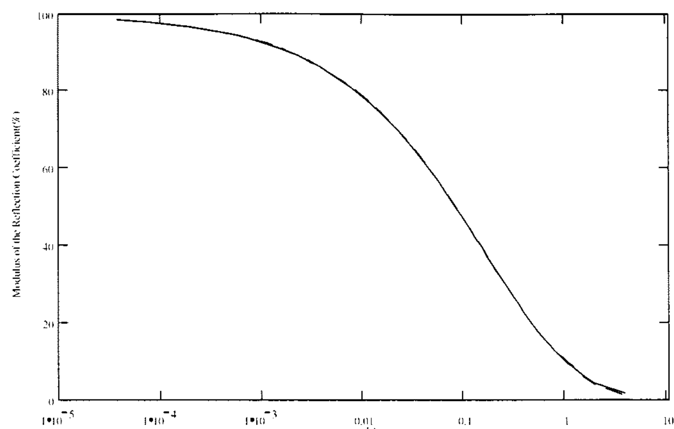


Fig. 7. Modulus of the reflection coefficient of the circular waveguide TE_{01} mode close to a cutoff cross section. Comparison between the values given by (17) (dashed line) and the mode-matching and generalized scattering-matrix method results (solid line).

The dependence of $|\rho_0|$ on the frequency difference participating in (6) is the same for all modes. The reflection coefficient for a fixed frequency difference is smaller for a smoother shape of a taper near its narrow end. The region where $|\rho_0|$ decreases from large to small values is narrowed for decreasing $v(L)$, where $v(L)$ has the order of tangent between the z-axis and the wall at the end of the taper (see Fig. 8). For example, two different circular waveguide tapers with different angles between their generating contour lines and the z-axis have been studied (see Fig. 9). If this angle is equal to 5°40′ (i.e., $v(L) = -0.1$), the reflection coefficient of the TE_{01} mode, $|\rho_0|$, varies from 1 at $t = 0$ to 0.5 at $t = -0.086$, for $[f - f_c]/f_c$ varying from 0 to 0.006, i.e., the operating frequency of the narrow waveguide increases by 0.6% from its cutoff frequency. If the angle is two times smaller, i.e., $v(L) = -0.05$, the same value of $|\rho_0| = 0.5$ is achieved for an operating frequency increment of 0.4% only,

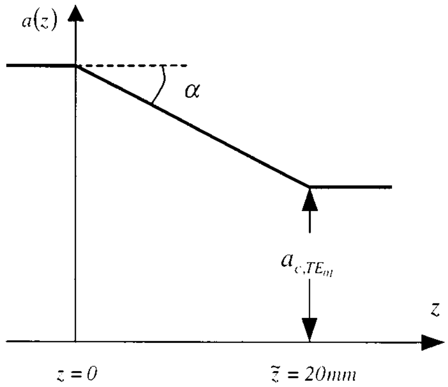


Fig. 8. Schematic showing the circular waveguide transition analyzed to study the frequency dependence of the modulus of the reflection coefficient as a function of the slope of the transition. The waveguide is axisymmetric with respect to the z-axis.

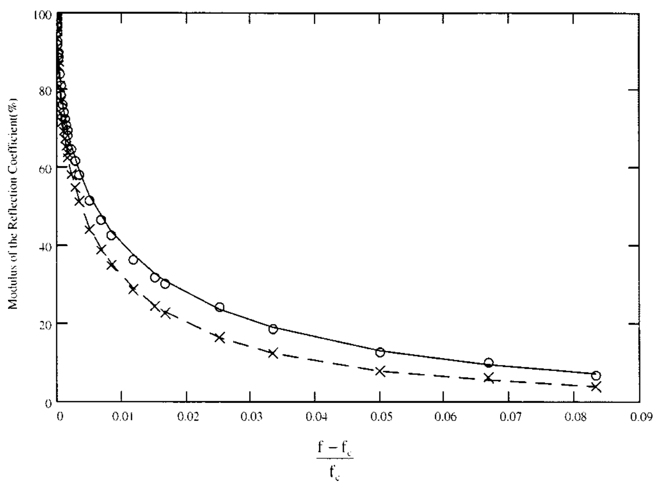


Fig. 9. Dependence of the modulus of the reflection coefficient close to a cutoff cross section on the frequency difference with the cutoff frequency. Comparison between the results given by (17) when the slopes of the linear taper are $v = -0.1$ (solid line) and $v = -0.05$ (dashed line) and the mode-matching and generalized scattering-matrix results (circles and crosses) for the same slopes.

and the frequency increment by 0.6% causes, in this case, a decrease of $|\rho_0|$ to 0.4.

V. Equivalent Boundary Condition for Parasitic Modes

In the paragraphs above, the reflection and transmission in the presence of a cutoff cross section of the propagating mode has been studied. Now there is a cutoff cross section of an excited parasitic mode at $z = \tilde{z}$, i.e., $h_j(\tilde{z}) = 0$, but the incident mode can propagate in the narrow waveguide (Fig. 10). Let us determine the amplitude of the parasitic mode propagating in the wide waveguide. For this problem, the solutions of (4) in the whole nonuniform waveguide will be required. We will assume in this section that the cutoff cross section is situated thus far from the beginning of the narrow waveguide and that the parasitic mode field is practically equal to zero at the beginning of the narrow waveguide.

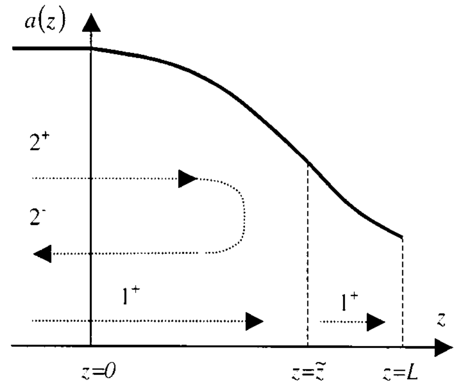


Fig. 10. Schematic showing a cutoff cross section of the parasitic mode located at $z = \tilde{z}$. The waveguide is axisymmetric with respect to the z-axis and the main and parasitic modes are depicted as "1" and "2," respectively.

According to (3a), far from the cutoff cross section, Q_j and R_j are related in the highest order terms of v by the following equation:

$$Q'_j + ih_jR_j = Q_m\Big(S_{jm} - S_{-jm}\Big). \tag{18}$$

To derive the algebraic relation between P_j and P_{-j}, i.e., the desired boundary condition, we express P_j and P_{-j} through Q_j and Q'_j and substitute (18) in them. Considering that the fundamental mode propagates without distortion, we obtain this "equivalent boundary condition"

$$\begin{aligned} &P_j(z)\exp\left(-i\left(\gamma_j - \tilde{\gamma}_j - \frac{\pi}{4}\right)\right) \\ &\quad - P_{-j}(z)\exp\left(i\left(\gamma_j - \tilde{\gamma}_j - \frac{\pi}{4}\right)\right) \\ &= \sqrt{\frac{h_m(0)}{h_j(z)}}\int_z^L \frac{\exp(-i\gamma_m)}{\sqrt{h_m}} \\ &\quad \times \Big\{S_{jm}\big(ih_jV + V'\big) + S_{-jm}\big(ih_jV - V'\big)\Big\}dz \end{aligned} \tag{19}$$

where z can have any arbitrary value in the region of large values of $|t|$, i.e., far enough from the cutoff cross section and V is proportional to the Airy function v in the vicinity of the cutoff cross section. In the region of large values of $|t|$, using the asymptotic expansion of the Airy functions, we obtain that this function has to be asymptotically defined at $z < \tilde{z}$ by

$$V(z) = h_j^{-(1/2)}\sin\left(-\gamma_j + \tilde{\gamma}_j + \frac{\pi}{4}\right)$$

and $z > \tilde{z}$ by

$$V(z) = \big(ih_j\big)^{-(1/2)}\exp\Big(-i\big(\gamma_j - \tilde{\gamma}_j\big)\Big).$$

The integrand contains such combinations of the coupling coefficients that have no singular points in the whole area of integration and particularly at the cutoff cross section. An equivalent boundary condition can be derived by that method for any other possible cases. If the mode j is a TM mode, (3b) must be employed near the cutoff cross section.

VI. Parasitic Mode Complex Amplitudes

The field of a parasitic mode can be derived from the first order coupled-wave (1) and from a boundary condition in the form (19) equivalent to the existing cross section. The second boundary condition is $P_j(0) = 0$, i.e., there is no incident mode of this kind.

Substituting the solution of these coupled wave equations for an arbitrary z outside of the cutoff area (and for $z < \tilde{z}$) into the boundary condition (19), we derive $P_{-j}(0)$, the desired amplitude of a parasitic mode propagating into the wide waveguide [1]

$$\begin{aligned} P_{-j}(0) &= \sqrt{\frac{h_m(0)}{h_j(0)}}\Bigg\{\exp\left(-2i\left(\tilde{\gamma}_j + \frac{\pi}{4}\right)\right) \\ &\quad \times \int_0^z S_{jm}\sqrt{\frac{h_j}{h_m}}\exp\Big(-i\big(\gamma_m - \gamma_j\big)\Big)dz \\ &\quad - \int_0^z S_{-jm}\sqrt{\frac{h_j}{h_m}}\exp(-i(\gamma_m + \gamma_j))dz \\ &\quad - \exp\left(-i\left(\tilde{\gamma}_j + \frac{\pi}{4}\right)\right)\int_z^L \frac{\exp(-i\gamma_m)}{\sqrt{h_m}} \\ &\quad \times \Big[S_{jm}\big(ih_jV + V'\big) \\ &\qquad + S_{-jm}\big(ih_jV - V'\big)\Big]dz\Bigg\} \qquad (20) \end{aligned}$$

where z can also have any value, but it cannot be near the cutoff cross section. Indeed, the factors S_{jm} and S_{-jm} in the last integral are identical to the corresponding factors in the first two integrals. Hence, the sum of all three integrals is an invariant in relation to z.

This representation has a quite definite physical meaning. The forward and backward propagating waves with index j are excited by the propagating m mode along each nonuniform waveguide section from $z = 0$ until the cross section z participating in (20). The superposition of all the parasitic modes propagating backward occurs at the cross section $z = 0$ and the result of that superposition is identical to the second term in (20). The parasitic modes propagating forward are incident on the cutoff cross section and reflected from it. They acquire the same additional phase factor as if this wave were incident from the outside of the nonuniform waveguide. In such a way, the first term appears in (20). Thereafter, the last term in (20) results from the parasitic wave field excited near the cutoff cross section. In this area, there is no sense to separate forward and backward propagating waves. The parasitic mode is excited there in a more complicated way.

The amplitudes of parasitic waves propagating forward are usually larger than the backward propagating wave amplitudes. Hence, if z in (20) is chosen not very distant from the cutoff cross section, the first term is the largest one and the third term must be taken into account by exceptional circumstances only. The proposed interpretation of (20) allows us to determine for other possible problems the two main terms in the field of the parasitic mode propagating into the wide waveguide without solving (4) and without deriving an equivalent boundary condition. For example, if the exciting wave is incident from the narrow waveguide situated at the left-hand side of the cutoff cross section, then the backward (and not forward) propagating waves have an additional phase factor corresponding to the parasitic wave reflection from the cutoff cross section.

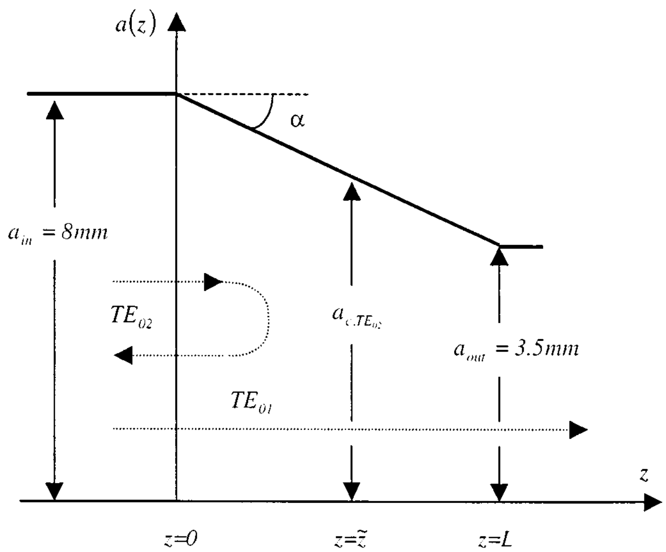


Fig. 11. Schematic showing the analyzed transition to obtain the dependence of the parasitic mode reflected power versus the slope of the transition ($\mathrm{tg}(\alpha) = v$). The waveguide is axisymmetric with respect to the z-axis, the input radius is 8 mm, the cutoff cross-section radius for the TE_{02} mode a_c is located at $z = \tilde{z}$ and the output radius is 3.5 mm.

VII. Case of Linear Tapers to a Cutoff Cross Section

We will apply the results above derived to waveguides where v is of the same order of magnitude for all z and is stepwise equal to zero at $z = 0^-$, i.e., to waveguides having a tilt of their contour lines. After some manipulations on (20) (see [1]), we obtain at $z = 0$

$$\begin{aligned} P_{-j}(0) = -i\left\{\frac{S_{jm}}{h_m - h_j}\right\}_{z=0}\exp\left(-2i\left(\tilde{\gamma}_j + \frac{\pi}{4}\right)\right) \\ + i\left\{\frac{S_{-jm}}{h_m + h_j}\right\}_{z=0} + I \quad (21) \end{aligned}$$

where the I term corresponds to the influence of the neighborhood of the intermediate cutoff cross section. This term is of lower order than the first two and it can be neglected for all problems where accurate computations are not needed (see [1]).

The first two terms in (21) have a simple physical meaning. The first term corresponds to the forward propagating waves reflected from the cutoff cross section and the second term to the backward propagating waves. From this point-of-view, the integral I in (21) can be explained as the influence of the neighborhood of the intermediate cutoff cross section.

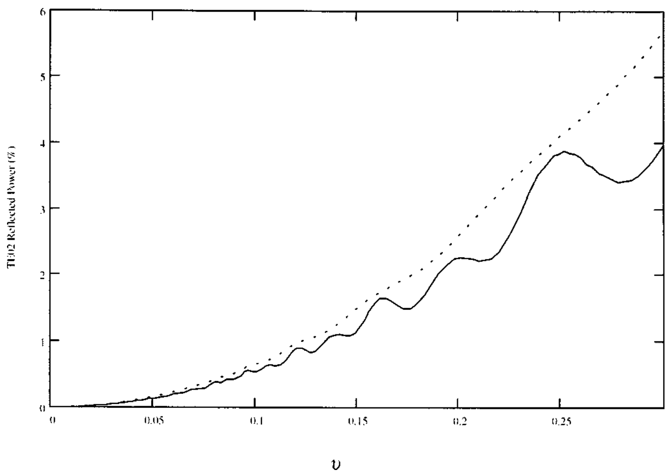


Fig. 12. Reflected power in the TE_{02} mode when it goes below cutoff in the transition shown in Fig. 11 versus the slope of the transition $(\mathrm{tg}(\alpha) = \upsilon)$. Comparison between the values given by (21) neglecting the I term (dashed line) and the mode-matching and generalized scattering-matrix method results (solid line).

In order to prove the validity of the results obtained with (21), a linear transition, where the input mode is the TE_{01} mode and the parasitic mode is the TE_{02} mode, has been studied at a frequency of 60 GHz. As can be seen in Fig. 11, both the input and output radii are fixed and the parameter that varies is L, varying the position of the cutoff cross section of the TE_{02} mode. For our example, it means that $\upsilon = (8 - 3.5)/L$. The dependence of the reflected TE_{02} power on this value has been studied.

Fig. 12 shows the results obtained using the mode-matching and generalized scattering-matrix methods compared to the ones given by (21) neglecting the I term. As can be seen, when the transition is not sufficiently smooth $(\upsilon \geq 0.2)$, the results are quite different, but for smooth enough transitions, they are in good agreement.

Thus, we have proven that the amplitude of a parasitic wave for waveguides with a contour line tilt depends only on values corresponding to the point of the tilt. As was mentioned above, this result does not contradict the nonlocal excitation of parasitic waves excited along the whole nonuniform waveguide.

VIII. Conclusions

In a cutoff cross section of the propagating mode, the values of the coupling coefficients are not small even for very slowly varying parameters. In order to use the coupled-wave equations and obtain solutions of the WKB type, we can use boundary conditions equivalent to the presence of the cutoff cross sections. The analytical expression of these boundary conditions in the cases of main and parasitic mode cutoff cross section have been tested using a mode-matching and generalized scattering-matrix code and the numerical integration of the coupled-wave equations; good agreement between the results were obtained in both cases.

References

[1] B. Z. Katsenelenbaum, L. Mercader, M. Pereyaslavets, M. Sorolla, and M. Thumm, *Theory of Nonuniform Waveguides—The Cross-Section Method*, ser. IEE Electromagn. Waves. London, U.K.: IEE Press, 1998, vol. 44.

[2] G. V. Kisunko, *Electrodynamics of Hollow Systems* (in Russian). St. Petersburg, Russia: VKAS, 1949.

[3] A. Stevenson, "General theory of electromagnetic horns," *J. Appl. Phys.*, vol. 22, pp. 1447–1454, 1951.

[4] S. A. Shelkunoff, "Conversion of Maxwell's equations into generalized telegraphist's equations," *Bell Syst. Tech. J.*, vol. 34, pp. 995–1043, 1955.

[5] F. Sporleder and H. G. Unger, *Waveguide Tapers, Transitions and Couplers*, ser. IEE Electromagn. Waves. London, U.K.: IEE Press, 1979, vol. 6.

[6] D. Hall, Ed., *Selected Papers on Coupled-Mode Theory in Guided-Wave Optics*. ser. SPIE Milestone. Bellingham, WA: SPIE, 1993, vol. MS 84.

[7] V. N. Manuilov, V. G. Starov, and V. G. Pavel'ev, "Method of plane cross-section in the theory of axisymmetrical structures, containing the pieces of irregular coaxial and circular waveguides," *Int. J. Infrared Millim. Waves*, vol. 18, no. 12, pp. 2323–2333, 1997.

[8] V. N. Manuilov and V. G. Pavel'ev, "The wave scattering on the step of the circular oversized waveguide filled in with insulator," *Int. J. Infrared Millim. Waves*, vol. 18, no. 12, pp. 2335–2342, 1997.

[9] V. V. Shevchenko, *Continuous Transitions in Open Waveguides—Introduction to the Theory*. Boulder, CO: Golem Press, 1971.

[10] T. Rozzi and M. Mongiardo, *Open Electromagnetic Waveguides*, ser. IEE Electromagn. Waves. London, U.K.: IEE Press, 1997, vol. 43.

[11] T. Itoh, Ed., *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*. New York: Wiley, 1989.

[12] H. Flügel and E. Kühn, "Computer-aided analysis and design of circular waveguide tapers," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 332–336, Feb. 1988.

[13] W. A. Huting and K. J. Webb, "Comparison of mode-matching and differential equation techniques in the analysis of waveguide transitions," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 280–286, Feb. 1991.

[14] ——, "Numerical analysis of rectangular and circular waveguide tapers," *IEEE Trans. Magn.*, vol. 25, pp. 3095–3097, July 1989.

[15] ——, "Numerical solution of the continuous waveguide transition problem," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1802–1808, Nov. 1989.

[16] F. Arndt, R. Beyer, J. M. Reiter, T. Sieverding, and T. Wolf, "Automated design of waveguide components using hybrid mode-matching/numerical EM building blocs in optimization-oriented CAD frameworks—state-of-the-art and recent advances," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 747–760, May 1997.

[17] P. Morse and H. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953.

[18] A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

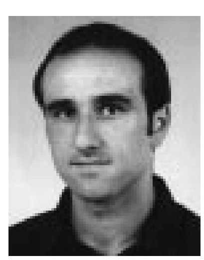
[19] W. C. Chew, *Waves and Fields in Inhomogeneous Media*, ser. Electromagn. Waves. Piscataway, NJ: IEEE Press, 1995.

[20] J. L. Doane, "Propagation and mode coupling in corrugated and smooth-wall circular waveguides," in *Infrared and Millimeter Waves*, J. K. Button, Ed. New York: Academic, 1985, pt. part IV, vol. 13, Millimeter Comp. Tech..

[21] H. Li and M. Thumm, "Mode conversion due to curvature in corrugated waveguide," *Int. J. Electron.*, vol. 71, no. 5, pp. 333–347, 1991.

[22] ——, "Mode coupling in corrugated waveguides with varying wall impedance and diameter changes," *Int. J. Electron.*, vol. 71, no. 5, pp. 827–844, 1991.

[23] S. E. Tsimring and V. G. Pavelyev, "The theory of nonhomogeneous electromagnetic waveguides containing critical cross-sections," *Radio Eng. Electron. Phys.*, vol. 27, no. 6, pp. 41–44, 1982.



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